Ontology and Mathematics

Outline

- Platonism
- Indispensability Argument
- Objections: Nominalism
- Primitive ontology and mathematical entities

Platonism

Platonism: realism towards the entities postulated in mathematical theories.

Two different takes on Platonism:

1. Mathematical statements are objectively true (they are not about mathematician's beliefs or true in virtue of sociological conventions).

NB: no mention of math entities is made here!

2. Mathematical statements are true in virtue of the existence of mathematical objects.

Conventional Platonism

- Conventional Platonism: every (consistent) mathematical theory actually describes a portion of the mathematical universe (as a physical theory is supposed to describe - part of - our physical universe).

- Rich ontology: every mathematical object that could possibly exist, does exist! (possibility and existence have the same extension). Ontological commitment towards all mathematical entities.

- Epistemological advantage: direct knowledge of mathematical entities (if an object even only mathematically conceivable does exist, we have direct access to it).

Structural Platonism

- Structural Platonism: structures and patterns are the subjects of mathematics (rather than entities).
- Slogan: "mathematical objects are places in structures".

- Are these structures instantiated?

a) Aristotelian position (*in re* structuralism): only instantiated structures do exist (really workably epistemology of math entities).

b) *Ante rem* structuralism: structures do not need to be instantiated to be considered real.

Fallibilism

- Mathematical objects are abstract entities: they do not have spatio-temporal properties;

- They lack causal power;

- Distinction certainty/a priori (internal disagreement among platonists):

Even though mathematics is a priori, it is not needed that it has to be certain: axioms often are conjectures (not self-evident truths).

- Conjectures in mathematics is not certain as it is in any other science.

Indispensability Argument

The Indispensability Argument (Quine - Putnam):

i. We should be ontologically committed to *all and only* the entities indispensable to our best scientific theories;
ii. Mathematical entities are indispensable to our best scientific theories;
iii. Therefore: we should be ontologically committed to the existence of mathematical objects.

Indispensable = necessary for explanation/ explanatorily essential

Indispensability Argument

- IA is strong argument in favor of Platonism;
- Quantification over mathematical entities: mathematical language is necessary to science;
- Epistemological equity: mathematical entities are seen (QP) to be epistemically on a par with theoretical entities in physical theories, since the belief of the existence of mathematical objects is justified by the evidence that confirms the theory as a whole (confirmational holism).
- NB: Mathematics is thus *empirically verified* as any other part of a given theory!!

Indispensability Argument

- IA as Inference to the Best Explanation (IBE);

- Scientific realism applied to mathematical entities:

Scientific realists appeal to the IBE to argue in favor of the existence of a given theoretical entity postulated in a certain physical theory.

IA has a similar structure: the best explanation to the applicability of mathematics to science is to postulate the existence of mathematical objects.

Indispensability Argument

- Baker's example of genuine mathematical explanation: the cicada's life cycle (from evolutionary biology).

- What is it to be explained?

i. The length of cicadas life cycle
ii. Two separate life-cycles durations in different geographical regions
iii. Periodic emergence of adult cicadas
iv. Synchronized emergence of adults cicadas
v. Prime-numbered-year cicadas life cycles lengths

Indispensability Argument

- i) and ii) explained by ecological constraints: poor availability of nutrients for nymphs in the soil and low temperature for great part of the year. (the souther we go, the hotter it gets!)

- iii) and iv) are explained in terms of biological laws: having a fixed periodic emergence is advantageous for mating opportunities (the same explanation of periodicity and synchronization)

- What about v)? Is there some evolutionary advantage to having a period that is prime?

Indispensability Argument

- Goles, Schulz and Marcus (GSM) explain this phenomenon in terms of predators' avoidance.
- They postulate a past period of cicadas when they were attacked by predators (themselves periodic) with a lower period.
- It is advantageous for cicadas to intersect as rarely as possible with predators.
- Then GSM conclude that the frequency of intersection is minimized when cicadas period is prime numbered:

A prey with a 12 year cycle will meet – every time it appears – properly synchronized predators appearing every 1, 2, 3, 4, 6 or 12 years, whereas a mutant with a 13 years period has the advantage of being subject to fewer predators (GMS 2001, p. 33 in Baker (2005), p. 231).

- This conclusion is based on theorems of number theory (for details see Baker (2005) 'Are there genuine mathematical explanations?').

Indispensability Argument

- The structure of the explanation:

1. Having minimized intersections with predators is evolutionarily advantageous (biological law)
2. Prime periods minimize intersections (number theoretic theorem)
3. Hence organisms with periodic life-cycles are likely to evolve periods that are prime (mixed biological and mathematical law).

If 3) is combined with the sentence "cicadas are limited by biological constraints to periods from 14 to 18 years", then it yields a specific prediction: *Cicadas are likely to evolve 17- years periods*.

- Baker: the purely mathematical component 2. is both essential and genuinely explanatory.

Naturalism and Holism

- IA is based upon naturalism and holism

i. Naturalism: science guides metaphysics in ontological enquiries. We ought to be ontologically committed to all and only the entities postulated by our best scientific theories

ii. Holism: scientific theories are confirmed *in toto*. If a theory is confirmed then the whole theory is, not single statements.

NB: naturalism and holism *taken together* justify the first assumption of IA.

Objections: Nominalism

- Nominalism: mathematical entities do not exist
- Example of nominalism: fictionalism (it does not exhaust all anti realists philosophies of maths)
- Fictionalism: mathematical statements are false in virtue of the (non) existence of mathematical objects.
- Simple epistemology of math entities: we do not know them, since they do not exist.
- Mathematical statements are true *within the fiction* of mathematics.
- Consistency and Ontological parsimony are required features for a mathematical theory. (from a fictional perspective not everything goes!)
- Problem: to explain the central role maths plays in science (applicability)

Objections

- Hartry Field: nominalization of mathematics
- Field rejects that maths is indispensable to science, he takes the dispensability of mathematics directly.

- In Field (1980) he rewrites (partially) the theory of Newtonian Gravitation without quantification over mathematical objects (nominalists' super hero). What is obtained is a reasonably attractive theory.

- Conservativeness of mathematics: math could be used in science since it is conservative.
- Pragmatic utility of mathematics.

Objections

- Penelope Maddy rejects the first assumption:

- Holism is inconsistent with the actual scientific practice: an holistic view of science is problematic since it cannot explain certain aspects of math and scientific practices. Ontological attitude of working scientists vary from belief to rejection.
- Naturalism respects the methods of the scientific community while holism seems to be inconsistent with it (rejecting differential support to entities of our best scientific theories). Then naturalism plus holism is inconsistent with actual scientific work.

- Maddy objections concern the methodological consequences of accepting naturalism and holism together.

Objections

- Maddy: working scientists do not take the applicability of math as a necessary indication of the existence of mathematical objects.
- Pragmatic approach to science: scientists use (regardless about ontological commitments) whatever mathematical theory is required w.r.t a given theory or experimental situation.
- Examples: idealizations and modeling.
- Elliot Sober: mathematics is not supported by empirical evidence.
- Common mathematical core shared by scientific theories.

Primitive ontology and mathematical entities

- Primitive Ontology Approach: division of the mathematical structure of any physical theory

i. Mathematical entities with a direct connection with physical real objects, called primitive variables;
ii. Mathematical structure used to implement the behavior of these primitive variables.

NB: we are ontologically committed to the physical entity described by a mathematical object, not necessarily to the mathematical entity itself.

Both realists and antirealists can support a PO approach to physical theories.

Primitive ontology and mathematical entities

- It seems that there is a *logical gap* between indispensability and reality;
- Arbitrariness of mathematical apparatus (e.g. Heisenberg vs. Schrödinger formulation of QM); (recall Maddy)

- *Weak objection*: non causal power of mathematical entities: they cannot play a effective explanatory role since they are non causally active; *Weakness: what about non causal explanations?*
- Stronger Objection: Do mathematical entities play effectively an explanatory role or, are the physical entities represented by these object responsible for the explanations?

Primitive ontology and mathematical entities

- Analogy: Math and Computer Programming
- Suppose that a theory is an algorithm with which we describe reality;
- Then there are several way to produce an "output" (solutions of dynamical equations) ⇒ using different mathematical strategies;
- But first we have to select the variables which will be implemented
- E.g. : two physical theories are physically equivalent iff they give the same histories for the PO, and PO is what remains invariant under physical equivalence;