Are there Genuine Mathematical Explanations of Physical Phenomena?

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Many explanations in science make use of mathematics. But are there cases where the mathematical component of a scientific explanation is explanatory in its own right? This issue of mathematical explanations in science has been for the most part neglected. I argue that there are genuine mathematical explanations in science, and present in some detail an example of such an explanation, taken from evolutionary biology, involving periodical cicadas. I also indicate how the answer to my title question impacts on broader issues in the philosophy of mathematics; in particular it may help platonists respond to a recent challenge by Joseph Melia concerning the force of the Indispensability Argument.

1. The Indispensability Argument

A central metaphysical debate in the philosophy of mathematics is between platonists, who postulate the existence of a realm of mind-independent, abstract mathematical objects, and nominalists, who deny the existence of such a realm. Much of the recent literature on the platonism–nominalism debate has focused on the pros and cons of the so-called 'Indispensability Argument'. Briefly stated, this argument claims that we ought rationally to believe in the existence of mathematical objects because we ought to believe our best available scientific theories, and quantification over mathematical objects is indispensable for science.

As it stands, the phrase 'indispensability for science' is vague. What, exactly, is the scientific purpose (or purposes) for which mathematics is supposed to be indispensable? Platonists typically sidestep this question by rephrasing the indispensability claim to state that our 'best' scientific theories quantify over mathematical objects. This style of response is holist, because the sole arbiter of ontological legitimacy is quantification by entire theories. More fine-grained questions concerning the theoretical role of individual posits are set aside, thus no analysis is required below the level of whole theories. This reliance on holism creates internal tensions within the platonist position. The origins of this

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holism component of indispensabilist platonism trace back to Quine, who was an arch-holist about confirmation. On his view, the various posits of a theory can only be confirmed *in toto*, and distinctions between the precise role of different posits do not matter. But not all platonists are holists, and it would be useful to have a version of the Indispensability Argument that did not rely so crucially on holism. Indispensabilist platonism also faces problems accounting for our apparent non-commitment to idealized concrete posits such as frictionless slopes, ideal gases, and infinitely-deep fluids.[1] Many of these posits play important—maybe even indispensable—roles in our best scientific theories; as it stands, the platonist lacks the resources to rule these ideal entities ontologically out of contention. If they are indeed indispensable then it seems we ought to believe in them. Finally, by staying at the level of whole theories, indispensabilist platonism fails to make manifest the *variety* of roles which mathematics plays in science. This makes it easier for the nominalist to dismiss mathematics as merely a calculational device or a descriptive shorthand.[2]

1.1 Indispensability and explanation

The Indispensability Argument has been recently criticized by the pro-nominalist Joseph Melia. These criticisms have sparked an exchange between Melia and the pro-platonist, Mark Colyvan, conducted over the course of three papers in recent issues of *Mind*. (Melia (2000), Colyvan (2002), Melia (2002)) One of Melia's targets is the issue of the role for which mathematics is purportedly indispensable. Melia's claim, in a nutshell, is that indispensability is not enough: mathematics must be indispensable *in the right way*. This requires the platonist to be more specific about the theoretical role which mathematics plays in science. Despite their opposing positions, Colyvan and Melia agree that establishing platonism stands or falls on whether specific examples can be found from actual scientific practice in which 'the postulation of mathematical objects results in an increase in the same kind of utility as that provided by the postulation of theoretical entities' (Melia 2002, p. 75).[3]

[1] Maddy refers to this as the 'Scientific Practice Objection' in Maddy (1992).

[2] See, for example, Balaguer (1998, p. 137) for one version of this 'narrow' characterization of mathematics.

[3] It needs to be the 'same kind' of utility to avoid begging the question against the nominalist; the platonist does not have to provide an independent defence of the ontological legitimacy of this kind of utility, if indeed it can be found, because the nominalist already appeals to it to justify her belief in electrons, quarks, and other theoretical concrete posits.

In this case, the mathematical postulates would have virtues that the nominalist has already conceded carry ontological weight.

One candidate for this kind of utility stands out, given the implicit endorsement of scientific realism by both the platonist and nominalist sides in the indispensability debate. A crucial plank of the scientific realist position involves inference to the best explanation (IBE) to justifies the postulation in particular cases of unobservable theoretical entities. Of course there are many philosophers who are not scientific realists, and alternative positions (notably constructive empiricism) are often based on a rejection of IBE in some or all cases. Nonetheless, the indispensability debate only gets off the ground if both sides take IBE seriously, which suggests that *explanation* is of key importance in this debate. It is no coincidence that both Colyvan and Melia explicitly endorse explanatory power.[4] And Hartry Field, one of the more influential recent nominalists, writes that the key issue in the platonism–nominalism debate is 'one special kind of indispensability argument: one involving indispensability *for explanations*' (Field 1989, p. 14).

We are interested, therefore, in cases where the postulation of mathematical objects yields explanatory power. A key strategic point of the indispensability-based approach is to focus on *external* applications of mathematics, since otherwise it is open to charges of circularity. Thus we shall not be discussing mathematical explanations of mathematical facts. And since our concern here is with the application of mathematics to science, the explanandum of any putative example must be some physical phenomenon. This brings us to my title question: are there genuine mathematical explanations of physical phenomena? The answer is of immediate relevance to the Indispensability Argument, and thereby to an important strand of the current debate between platonists and nominalists over the existence of abstract mathematical objects.

Both Colyvan and Melia attempt to tip the debate in their favour, Colyvan by presenting some alleged examples of mathematical explanations in science, and Melia by arguing on general grounds against the likelihood (or even possibility) of any such examples. I shall begin by reviewing of each of their efforts.

1.2 Colyvan's example of mathematical explanation

Colyvan is sympathetic to the view that pure mathematics can be genuinely explanatory with respect to physical phenomena. His strategy for

[4] '[T]here's no doubt that explanatory power is a theoretical virtue (at least for scientific realists).' (Colyvan 2002, p. 72); '*explanatory* power of a theory is also a virtue of a theory … .' (Melia, 2002, p. 75)

bolstering this claim is to present various examples from scientific practice. Consider the following meteorological example.

> We discover that at some time t_0 there are two antipodal points p_1 and p_2 on the earth's surface with exactly the same temperature and barometric pressure. What is the explanation for this coincidence? (Colyvan 2001, p. 49)[5]

Colyvan claims that there are actually two coincidences which need to be explained;

(1) Why are there *any* such antipodal points?

(2) Why p_1 and p_2 in particular?

He argues that there is a purely causal story which can (in principle) be told to explain (2), based on the detailed prior histories of the weather patterns in the vicinity of p_1 and p_2. However this causal story fails to explain (1). As Colyvan puts it, the causal story 'does not explain why p_1 and p_2 have the *same* temperature and barometric pressure, just why each has the particular temperature and pressure that they have, and that these *happen* to be the same' (Colyvan 2001, p. 49). The explanation for this sameness lies in a corollary of the Borsuk-Ulam theorem, from algebraic topology, which implies that there are always antipodal points on the earth's surface which have the same temperature and barometric pressure. Colyvan claims that the proof of this theorem provides the missing part of the explanation of (1).

I have no quibbles with the basic structure of Colyvan's example, but I have reservations about whether it is genuinely explanatory. First, the chances of actually finding two such antipodal points are presumably very remote; they are not something that we are likely to stumble across by accident. Nor are they something that meteorologists would search for unless they already knew about the result of the mathematical theorem. In other words, the explanandum would probably not suggest itself to us unless and until we had the explanans in hand. If this is right, then what Colyvan presents here, at least *prima facie*, is not an explanation but a prediction. Secondly, even if meteorologists did discover two such antipodal points, would they consider this a phenomenon that was in need of explanation? Colyvan suggests that a criterion for adequate explanation is that it 'must make the phenomena being

[5] It should be noted that Colyvan offers this example in the context of arguing for the possibility of *noncausal* explanations. However it is clearly also relevant to answering Melia's challenge.

explained less mysterious.' (Colyvan 2001, p. 47) I am doubtful whether in this case there really a mystery here to be reduced in the first place.[6]

1.3 Melia's counterargument

Two of Colyvan's other examples of putative mathematical explanation are from the theory of relativity and each refers to the geometry of Minkowski space-time. The first example involves the bending of light near to massive bodies, and the second example involves the FitzGerald-Lorentz contraction of moving bodies along their direction of motion. Melia is unconvinced by these examples. His principal objection is that, even if the mathematical apparatus is indispensable, its role in these and similar cases is merely to pick out or 'index' efficacious physical objects or properties;

> [W]hen we come to explain [physical fact] *F*, our best theory may offer as an explanation '*F* occurs because *P* is $\sqrt{2}$ metres long'. But we all recognize that, though the number $\sqrt{2}$ is cited in our explanation, it is the *length* of *P* that is responsible for *F*, not the fact that the length is picked out by a real number. (Melia 2002, p. 76)

> T_2 expresses the fact that *a* is 7⁄11 meters from *b* by using a three place predicate relating *a* and *b* to the number 7⁄11, nobody thinks that this fact holds *in virtue* of [this relation]. Rather, the various numbers are used merely to index different distance relations, each real number corresponding to a different distance. (Melia 2000, p. 473)

One interpretation of the above passages is that the mathematical apparatus in these cases is not genuinely explanatory because the role of the numbers $\sqrt{2}$ and 7⁄11 is *arbitrary*. The charge of arbitrariness is correct up to a point, since facts about which specific numbers figure an the explanation of a given physical fact are relative to a (more-or-less) arbitrary choice of units. However the platonist could argue that the level of focus here is too fine-grained. She is not arguing that individual numbers are indispensable for science or play an explanatory role in science, but rather that certain mathematical *theories* are indispensable and explanatory. Thus in the above examples it may be the case that quantification over the real numbers is necessary even though quantification specifically over $\sqrt{2}$, for example, is not.[7] Arbitrariness of this sort is a

[6] It is ironic, given this worry, that Colyvan uses the term 'coincidence' in describing his example. In everyday parlance, coincidences are phenomena for which *no* further explanation is required.

[7] It might be objected that even this claim is too strong, since all that is really necessary is some mathematical structure *isomorphic* to the real numbers, for example an appropriately chosen collection of sets. For more on how issues of multiple reducibility in mathematics impact the Indispensability Argument see Baker (2003).

general feature of geometrically-based examples of mathematical application, a feature common to all of Colyvan's examples.

Nor is arbitrariness the only problem with geometrically based cases. Another problem arises from the ambiguity in the subject matter of geometry, in particular whether this subject matter is mathematical or physical. Individual geometrical terms such as 'triangle' may refer either to mathematical or to physical objects, and the historical trajectory of Euclidean geometry, from descriptor of physical space to free-standing formal system, shows a similar bridging of the mathematics/physics boundary at the level of geometrical theories. This is one reason why nominalists often object that geometrical explanations are not genuinely mathematical. And it suggests that we should look elsewhere than geometry for a convincing case of mathematical explanation in science

A second interpretation of Melia's 'indexing' remarks is that reference to $\surd 2$ or $7/11$ is not explanatory in the above examples because these numbers are *acausal*. If all explanation (at least of physical phenomena) is causal explanation, then this yields a quite general objection to the possibility of combining a platonist view of mathematics with the thesis that there are genuine mathematical explanations of physical phenomena. This interpretation fits with Melia's remarks in other passages which suggest that his underlying problem with putative examples of mathematical explanation stems from mathematical objects lacking any causal role. He argues, for example, that what distinguishes the postulation of quarks from the postulation of mathematical entities is that in the former case 'the complex objects *owe their existence* to these fundamental objects' (Melia 2000, p. 474). However, I do not think that Melia means to rule out the possibility, at least in principle, of noncausal explanations of physical phenomena, for his remarks elsewhere seem to leave this issue as an open question to be settled by empirical investigation, as for example when he writes that 'there *may* be applications of mathematics that do result in a genuinely more attractive picture of the world' (Melia 2000, p. 474).[8] Perhaps there is still *some* room for Melia to squeeze out of this verdict, for example by arguing that the sorts of utility which mathematics may possess do not include explanatory power. But if this were his intention then it would be obtuse of him not to state it explicitly. Moreover, even if Melia does decide to take a stand against the possibility of noncausal explanation, this is a separate

[8] He also states that it is 'only by a careful analysis of the uses to which mathematics is put [that we will] be able to judge whether or not the indispensability argument supports Platonism' (Melia 2002, p. 76).

issue, one that is far from settled, and substantive argument would be needed to support this position. In other words, it seems that the 'no noncausal explanation' thesis is not one to which the nominalist can appeal without begging some pivotal questions.

The Melia–Colyvan debate, it seems fair to conclude, has not been conclusively resolved by either side. Colyvan has not come up with any unequivocal cases of mathematical explanation in science, and Melia has not given any non-question-begging grounds for thinking that such explanations are impossible, or even unlikely. My goal in the next section is to settle the debate in Colyvan's favour by presenting a detailed example of a genuinely explanatory application of mathematics to science.

2. Case study: periodical cicadas

My principal case study, featuring a genuinely mathematical explanation of a physical phenomenon, is drawn from evolutionary biology. Its subject is the life-cycle of the so-called 'periodical' cicada. North America is home to several species of cicada, large fly-like insects (often erroneously referred to as 'locusts') notable for the shrill calls they produce by rubbing their wings against their bodies. Three species of cicada of the genus *Magicicada* share the same unusual life-cycle. In each species the nymphal stage remains in the soil for a lengthy period, then the adult cicada emerges after either 13 years or 17 years depending on the geographical area. Even more strikingly, this emergence is synchronized among all members of a cicada species in any given area. The adults all emerge within the same few days, they mate, die a few weeks later and then the cycle repeats itself.

2.1 What needs to be explained?

Biologists have long found features of the life-cycle of periodical cicadas mysterious, and this is reflected both in the substantial literature devoted to this topic and in biologists' specific remarks.[9] There are at least five distinct features of this life-cycle for which explanations have been sought by biologists;

(i) The great duration of the cicada life-cycle.

(ii) The presence of two separate life-cycle durations (within each cicada species) in different regions.

[9] For example, that '[p]eriodical cicadas are among the most unusual insects in the world. (Yoshimura 1997, p. 112)

(iii) The periodic emergence of adult cicadas.

(iv) The synchronized emergence of adult cicadas.

(v) The prime-numbered-year cicada life-cycle lengths.

Features (i) and (ii) concern the temporal range of the life-cycle. Biologists have argued that the long life-cycle of *Magicicada* is due both to the poor availability of nutrients for nymphs, and to the low soil temperatures for much of the year. Together these environmental stresses force nymphs to spend several years maturing into adults. Each of these negative factors is less pronounced in the southern regions of *Magicicada*'s range. Hence it is not surprising that life-cycle lengths are shorter for each species in the southern parts of the U.S. In short, both (i) and (ii) seem explicable in terms of specific ecological constraints.

Features (iii) and (iv) concern coordination of the life-cycles of different individuals. Given that cicada nymphs require several years to develop into adults, and that the adult stage is very brief, having a fixed periodic emergence is advantageous in terms of maximizing mating opportunities. It ensures that the offspring of a particular mating generation will all appear at the same time, several years down the line. Synchronization makes sense for the same reason. Especially in areas which can support only a sparse population of cicadas, staggering different subpopulations to emerge at different times may produce so few adults at any one time that it is difficult to find mates. These explanations of (iii) and (iv) rely on (evolutionary) biological 'laws' which potentially apply to any organism with a long life-cycle and brief adult stage.

2.2 Explaining prime cycles

This leaves feature (v) to be explained, and with it one key question to be answered: why are the life-cycle periods *prime*? In other words, given a synchronized, periodic life-cycle, is there some evolutionary advantage to having a period that is prime? If so, this would help explain why 13 and 17 are the favoured cycle periods for each of the three species of the genus *Magicicada*. In seeking to answer this question, biologists have come up with two basic alternative theories.

An explanation of the advantage of prime cycle periods has been offered by Goles, Schulz and Markus (2001) (henceforth, GSM) based on avoiding predators. GSM hypothesize a period in the evolutionary past of *Magicicada* when it was attacked by predators that were themselves periodic, with lower cycle periods. Clearly it is advantageous—

other things being equal—for the cicada species to intersect as rarely as possible with such predators. GSM's claim is that the frequency of intersection is minimized when the cicada's period is prime;

> For example, a prey with a 12-year cycle will meet—every time it appears—properly synchronized predators appearing every 1, 2, 3, 4, 6 or 12 years, whereas a mutant with a 13-year period has the advantage of being subject to fewer predators. (Goles et al. 2001, p. 33)

A second explanation, proposed by Cox and Carlton and by Yoshimura, concerns the avoidance not of predators but of hybridization with similar subspecies. (Cox and Carlton 1988, 1998, Yoshimura 1997). A crucial factor for periodical insects is to have sufficient mating opportunities during their brief adult stage. Almost as important, however, is to avoid mating with subspecies that have different cycle periods to their own. For example, if some of a (hypothetical) population of synchronized 10-year cicadas were to mate with some 15-year cicadas then their offspring would likely have a period of around 12 or 13 years. These hybrid offspring would emerge well after the next cycle of the 10-year cicadas and hence their mating opportunities would be severely curtailed.[10] Yoshimura considers a putative stage in the evolutionary past where there were several subspecies of cicada with periods in the 14- to 18-year range. He shows how 17-year cicadas would intersect least often with cicadas of other periods in this range. Yoshimura explicitly connects these results to the fact that 17 is a prime number.[11]

2.3 A shared number theoretic basis

The mathematical underpinnings of both the predation and the hybridization explanations lie in number theory, the branch of mathematics which investigates the often deep and subtle relationships between the integers. The mathematical link between primeness and minimizing the intersection of periods involves the notion of *lowest common multiple* (lcm). The lcm of two natural numbers, m and n, is the smallest number into which both m and n divide exactly; for example, the lcm of 4 and 10 is 20. Assume that m and n are the life-cycle periods (in years) of two subspecies of cicada, C_m and C_n. If C_m and C_n intersect in a particular year, then the year of their next intersection is given by the lcm of m and n. In other words, the lcm is the number of

[10] A further disadvantage is that hybrid offspring are likely to differ even among themselves with respect to life-cycle length.

[11] Yoshimura (1997), p. 115

years between successive intersections. In fact the fundamental property in this context is not primeness but *coprimeness*; two numbers, *m* and *n*, are coprime if they have no common factors other than 1 (i.e. neither number is divisible by the other). All that is needed to underpin the above explanations are the following two number-theoretic results.

> **Lemma 1**: the lowest common multiple of *m* and *n* is maximal if and only if *m* and *n* are coprime.[12]

Lemma 1 implies that the intersection frequency of two periods of length *m* and *n* is maximized when *m* and *n* are coprime. We get from coprimeness to primeness *simpliciter* with a second result;

> **Lemma 2:** a number, *m*, is coprime with each number $n < 2m$, $n \neq m$ if and only if *m* is prime.

The mathematics for the predation explanation is already contained in the above two Lemmas. Predators are assumed to have relatively low cycle periods. It therefore suffices to show that prime numbers maximize their lcm relative to all lower numbers. More formally, we need to show that for a given prime, *p*, and for any pair of numbers, *m* and *n*, both less than *p*, the lcm of *p* and *m* is greater than the lcm of *n* and m. But this follows directly from Lemmas 1 and 2. Furthermore, only prime numbers maximize their lcm's in this way, so in this respect primes are optimal.

Hybridization is assumed to occur only between subspecies with similar period lengths. The range of alternative periods that count as 'similar' will depend on the biological details—Yoshimura assumes a range of 4 or 5 years in the case of cicadas. It therefore suffices for the mathematical underpinnings of the hybridization explanation to establish that prime numbers maximize their lcm relative to other 'nearby' numbers. This follows immediately from Lemmas 1 and 2. Consider a prime, *p*, and a range $[p-r, p+r]$. Assuming $r < p$,[13] the other numbers in the range are all between 1 and 2*p*, hence *p* is coprime with all of them, from Lemma 2.

2.4 The structure of the explanation

The basic structure common to the predation and hybridization explanations is as follows;

[12] For proofs of all lemmas, see Landau (1958).

[13] This implies that the range is 'small' with respect to *p*.

(1) Having a life-cycle period which minimizes intersection with other (nearby / lower) periods is evolutionarily advantageous. [**biological 'law'**]

(2) Prime periods minimize intersection (compared to non-prime periods).
[**number theoretic theorem**]

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(3) Hence organisms with periodic life-cycles are likely to evolve periods that are prime. [**'mixed' biological / mathematical law**]

When the law expressed in (3) is combined with

(4) Cicadas in ecosystem-type, E, are limited by biological constraints to periods from 14 to 18 years.[14][**ecological constraint**]

it yields the specific prediction

(5) Hence cicadas in ecosystem-type, E, are likely to evolve 17-year periods.

What we have, then, is a five-step argument which—through plugging in the different ecological constraints in step (4)—aims to explain the phenomenon of cicadas having 13- and 17-year periods. The explanation makes use of specific ecological facts, general biological laws, and number theoretic results. My claim is that the purely mathematical component, (2), is both essential to the overall explanation and genuinely explanatory in its own right. In particular, it explains *why* prime periods are evolutionarily advantageous in this case. Thus (in the terms used by Melia and Colyvan) this application of mathematics yields 'explanatory power'.

3. Is the cicada example a genuinely mathematical explanation?

The cicada example is only helpful to the platonist position if it meets the three conditions mentioned in sections 1.1 and 1.2. The first condition is that the application be external to mathematics. It is clearly met by this example since the phenomenon being explained—the period length of cicadas—lies outside the realm of pure mathematics. The second condition is that the phenomenon in question must be in need of explanation. This condition is met, since the life-cycle of periodical

[14] Clearly a parallel constraint may be formulated for 13-year cicadas, in which the ecosystem limits potential periods to the range from 12 to 15 years.

cicadas is considered 'remarkable' and 'mysterious' by biologists themselves, as is evidenced by the quotes cited earlier in the paper. A third condition for genuine explanation is that the phenomenon must have been identified independently of the putative explanation (otherwise it is more like a prediction). This is true in the cicada case. Cicadas with 13- and 17-year cycles were known prior to any explanation involving primeness, indeed they were discovered over 300 years ago, well before the development of number theory as a freestanding branch of mathematics.

There seems to be little doubt, therefore, that the cicada example is a case of genuine explanation, and that it involves reference to mathematical objects. But is it a genuine *mathematical* explanation? Melia, in his discussion of Colyvan's example involving Minkowski space-time, raises the following objection.

> True, when we come to give the geometric explanation of a certain relativistic fact, we may find ourselves indispensably using mathematical objects. But it doesn't follow from this that mathematical objects play a part in the explanation itself, or add to the explanatory power of the theory … (Melia 2002, p. 76)

I do not see how one can coherently deny that mathematical objects *play a part* in the explanation. However, it does not follow from this that all explanations involving mathematics are *ipso facto* mathematical explanations, and this is how I read the second part of Melia's worry.

What needs to be checked in the cicada example, therefore, is that the mathematical component of the explanation is explanatory in its own right, rather than functioning as a descriptive or calculational framework for the overall explanation. This is difficult to do without having in hand some substantive general account of explanation. The philosophical analysis of explanation is itself a thorny issue (and not one we shall attempt to settle here), but it may be useful to canvas the three leading contemporary philosophical accounts of explanation—the causal account, the deductive-nomological account, and the pragmatic account—to see if any of them can fruitfully be applied in the present context.

According to the causal account, explaining a phenomenon involves giving a description of its various causes. Clearly this account is incompatible with the existence of *any* genuine mathematical explanations, since mathematical objects (if they exist) are acausal. Hence this account is not helpful for the current debate since to adopt this account is effectively to beg the question against the platonist.[15]

[15] This same point arose in section 1.3 in response to certain of Melia's remarks.

According to the deductive-nomological account, explaining a phenomenon involves constructing an inference of the phenomenon from premises which include statements of general laws of nature. The layout of the cicada explanation in section 2.4 has this form. But does the deductive-nomological model have the resources to distinguish explanatory from non-explanatory *components* of explanations? One point in the platonist's favour is that the purely mathematical premiss (2) of the cicada inference is in the form of a general law, in this case a theorem of number theory. A broadening of the category of laws of nature to include mathematical theorems and principles, which share commonly cited features such as universality and necessity, would count the mathematical theorem (2) as explanatory on the same grounds as the biological law (1).

According to the pragmatic account, explaining a phenomenon involves providing an answer to a 'why'-question which shows how the phenomenon is more likely than its alternatives. This is the sketchiest of the three accounts, but perhaps also the most useful in the present context. It suggests that genuinely explanatory applications of mathematics ought to be reconfigurable as answers to questions about why a certain physical phenomenon occurred. This parallels cases of explanation involving concrete theoretical posits, which are unproblematic common ground for both platonists and nominalists in the indispensability debate. Why is the light from certain distant galaxies getting bent? Because there is a black hole between us and the distant galaxies. Why do periodical cicadas have prime periods? Because prime numbers minimize their frequency of intersection with other period lengths. In each case we have a naturally motivated why-question paired up with a (partial) answer. And in each case the answer seems genuinely explanatory.[16]

It is time to take stock. We have surveyed three accounts of explanation. The two of these accounts which allow for the *possibility* of mathematical explanations both support the claim that the cicada case study is an example of a genuinely explanatory application of mathematics to science. The third, causal account of explanation rules out the possibility of mathematical explanations; however this account is problematic for independent reasons. Finally, the alleged explanatoriness of the number theoretic component of the cicada case study seems to mesh

[16] It should be noted that advocates of the pragmatic account of explanation, for example van Fraassen, often combine this account with a broader anti-realist stance which rejects unrestricted use of inference to the best explanation. However, there seems to be no reason why the pragmatic account cannot instead be combined with versions of realism.

well both with our intuitions and with the intuitions expressed by biologists working in this area.

The nominalist is not without potential challenges to the explanatoriness of this case study. However, I think that the nature of the cicada example blocks several of the more promising lines of objection. As we saw in section 1.3, Melia's favoured objection—for instance to Colyvan's Minkowski space-time examples—is that the mathematical apparatus merely serves to pick out or 'index' efficacious physical objects and properties. This boils down to the allegation that the role of mathematical objects in such cases is *arbitrary*. The platonist can concede that arbitrariness of this sort is a frequently encountered feature, especially of geometrically-based examples of mathematical application. However, since the cicada example is based on number theory rather than on geometry, it is better placed to meet this objection. There is nothing arbitrary about the role of 13 and 17 in this case. The units in which periods are measured, namely years, are not chosen on an *ad hoc* basis but are rooted in the physical features of the example. And the mathematical explanation makes reference to a specific feature of 13 and 17, namely their primeness, which is not possessed by arbitrary integers.

The cicada example is also well-placed to meet a second objection to geometry-based applications of mathematics, also mentioned in section 1.3. This stems from the issue of the ambiguity in the subject matter of geometry, in particular whether this subject matter is mathematical or physical. By contrast with geometry, number theory deals with the intrinsic mathematical properties of the natural numbers, and as such is an archetype of a pure mathematical theory. Indeed there is ongoing fascination, especially among mathematicians, with the fact that number theory is applicable *at all*.[17]

5. Conclusions

I have argued that there are genuine mathematical explanations of physical phenomena, and that the explanation of the prime cycle lengths of periodical cicadas using number theory is one example of such. If this is right, then applying inference to the best explanation in the cicada example yields the conclusion that numbers exist.

Whatever cases of putative mathematical explanation the platonist might come up with, there will always be some leeway for nominalist objections since the role of mathematical posits is unlikely ever to *exactly* match the role of concrete unobservables, such as electrons.

[17] See, for example, Schroeder (1992) on the 'unreasonable effectiveness' of number theory.

Nonetheless, I think that the cicada example bolsters the platonist position both internally and externally. From an internal perspective, the platonist has reduced the reliance of the Indispensability Argument on holism, thus allowing for a potential distinction to be drawn between mathematical posits and idealized concrete objects such as frictionless slopes and perfect spheres.[18] The external benefit of focusing on the cicada example (and doubtless other examples with similar force can be found) is that it shifts the burden of proof onto the nominalist. Mathematics—specifically number theory—plays a genuinely explanatory role in accounting for the cycle lengths of periodical cicadas. The mathematical apparatus in this case in neither arbitrary nor is it straightforwardly dispensable. It therefore presents a challenge to any nominalist, such as Melia, who leaves open the possibility of genuine mathematical explanations of physical phenomena.

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[18] My hunch is that reference to idealized concrete objects may provide a descriptive framework for scientific theorizing, but that such reference is not genuinely explanatory.

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